Review Problems For Test # 1

Problem 1. *Find the area of the triangle whose vertices are the points* $A = (2, 1, 2)$, $B = (-1, 2, 0)$, *and* $C = (0, -1, 3)$.

Problem 2. *Find the volume of the parallelepiped formed by vectors* $\vec{A} =< 2, 1, 0 >$, $\vec{B} =< 3, 2, -1 >$, *and* $\vec{C} =< -1, 0, 2) >$.

Problem 3. *Determine whether the following three vectors are in the same plane:* $\vec{A} =< 3, 1, -1 >$, $\vec{B} =< 2, -2, 4 >$, *and* $\vec{C} =< 7, 5, -7) >$.

Problem 4. *Write equation of the straight line that passes through the point* $P = (3, 1, 2,)$, *and is perpendicular to the lines* $x = 1 + 2t, y = -1 + -2t, z = 3t$, *and* $\dfrac{x+1}{-2} = \dfrac{y+15}{2} = \dfrac{z+23}{-1}$.

Problem 5. *Write equation of the straight line that passes through the point* $P = (2, 1, -1)$, *and is perpendicular to the plane* $3x + 2y + z = 5$.

Problem 6. *Show that the line containing the points* $P = (0, 2, -2)$ *and* $Q = (2, -1, 1)$ *is perpendicular to the line with equation* $\dfrac{x-10}{3} = \dfrac{y+15}{4} = \dfrac{z-20}{2}$.

Problem 7. *Write equation of the plane containing the point* $P = (0, 2, -1)$ *and the line* $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-2}{5}$.

Problem 8. *Show that the following two planes* $x + 2y - 3z = 5$, *and* $7x + 4y + 5z = 1$ *are perpendicular.*

Problem 9. *Does the line* $\dfrac{x-2}{3} = \dfrac{y-3}{2} = \dfrac{z-1}{-8}$ *intersect the plane* $4x - 2y + z = 3$ *? If it does, find the point of intersection.*

Problem 10. *Find the point of intersection of the line* $\dfrac{x-3}{3} = \dfrac{y-1}{2} = z + 1$, *and the plane* $2x - 3y + z = 2$.

Problem 11. *Consider the two planes* $2x - y + z = 2$, *and* $x - 2y + z = 1$.

- **(a)** *Find the line of intersection of these two planes.*
- **(b)** *Write equation of the plane through the point* $P(1, 2, 1)$ *that is perpendicular to the line of intersection of these two planes.*

Problem 12. *Find equation of the plane that contains the lines* $\dfrac{x-1}{5} = \dfrac{y-2}{2} = z+1$ *and* $\dfrac{x-2}{-1} = \dfrac{y+1}{3} = \dfrac{z-2}{-3}$.

Problem 13. *Find the distance between the line with equation* $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-2}{5}$, *and the line with parametric equation* $x = 3 + 2t$, $y = 3t$, $z = 1 + 5t$.

Problem 14. *Find the distance between the planes* $2x + y - z = 3$, *and* $2x + y - z = 9$.

Problem 15. *Find the distance between the line* $\dfrac{x+1}{3} = y + 3 = \dfrac{z+2}{4}$, *and the plane* $2x - 2y - z = 1$.

Problem 16. *Find the distance between the two lines* $\dfrac{x-1}{2} = y - 2 = \dfrac{z+1}{3}$, *and* $\dfrac{x}{3} = \dfrac{y-1}{-2} = \dfrac{z-2}{2}$.

Problem 17. *Write equation of the sphere that is centered at the point* $C = (1, 2, 1)$, *and is tangent to the plane* $2x - 2y - z = 3$.

Problem 18. *Find the center and radius of the sphere with equation* $x^2 + y^2 + z^2 - 4x + 2y - 6z = 22$.

Problem 19. *Write equation of the sphere one of whose diameter has two end points* $P = (3, 1, 2,)$, *and* $Q = (1, 5, 0)$.

Problem 20. *Sketch and identify the following surfaces:*

(a) $y = x^2 + 1$.
(b) $z = 4x^2 + y^2$.
(c) $z = \sqrt{x^2 + y^2}$.
(d) $z^2 = x^2 + 4y^2$.
(e) $z = 4 - x^2 - y^2$.
(f) $x^2 + 4y^2 + 16z^2 = 16$.

Problem 21. *Let* $\vec{\mathbf{F}}(t) = \ln(t+1)\,\vec{\mathbf{i}} + \dfrac{\sin 3t}{t}\,\vec{\mathbf{j}} + \sqrt{5-4t}\,\vec{\mathbf{k}}$ *be a vector-valued function. Then*

(a) *Find the domain of* $\vec{\mathbf{F}}$.
(b) *Find* $\lim_{t\to 0} \vec{\mathbf{F}}(t)$.
(c) *Find* $\dfrac{d}{dt}\vec{\mathbf{F}}(t)$.

Problem 22. *Let* $\vec{\mathbf{F}}(t) = \ln\left(\dfrac{2t-1}{t+1}\right)\vec{\mathbf{i}} + \dfrac{2\sin 3t}{t}\,\vec{\mathbf{j}} + \dfrac{5t^2-2t}{t^2-4}\,\vec{\mathbf{k}}$ *be a vector-valued function. Then*

(a) *Find the domain of* $\vec{\mathbf{F}}$.
(b) *Find* $\lim_{t\to\infty} \vec{\mathbf{F}}(t)$.
(c) *Find* $\dfrac{d}{dt}\vec{\mathbf{F}}(t)$.

Problem 23. *Find* $\dfrac{d}{dx}\vec{F}(x)$, *where* $\vec{F}(x) = \left(\displaystyle\int_0^x \sqrt[3]{1+t^3}\,dt\right)\vec{i} + (x\cos^2 x)\vec{j} + \left(\displaystyle\int_{10x}^{x^2} \sqrt[3]{1+t^3}\,dt\right)\vec{k}$.

Problem 24. *Find the length of the curve* $\mathbf{r}(t) = 3t\cos t\,\mathbf{i} + 3t\sin t\,\mathbf{j} + 4t\,\mathbf{k}$ *when* $0 \le t \le 4$.

Problem 25. *Let* $\mathbf{r}(t) = e^t\,\mathbf{i} + e^{-t}\,\mathbf{j} + \sqrt{2}\,t\,\mathbf{k}$ *be a parametrization of a curve* C. *Find the tangent, normal, and curvature of* C.

Problem 26. *Find tangent, normal, and the curvature of the curve traced out by* $\mathbf{r}(t) = \frac{1}{3}(1+t)^{3/2}\,\mathbf{i} + \frac{1}{3}(1-t)^{3/2}\,\mathbf{j} + \frac{\sqrt{2}}{2}t\,\mathbf{k}$.