Review Problems For Test # 2

Problem 1. *The acceleration of a moving object is given by* $\vec{a}\,(t) = \langle 12t,\ 2,\ 6t+4\rangle$. *Find the position vector of this object with the initial conditions* $\vec{v}\,(0) = \langle 1,\ 2,\ 1\rangle$ *and* $\vec{r}\,(0) = \langle 1,\ -2,\ -1\rangle$.

Problem 2. *The acceleration of a moving object is given by* $\vec{a}\,(t) = \langle 6,\ 4-12t,\ 24t\rangle$. *Find the position vector of this object such that* $\vec{v}\,(1) = \langle 1,\ 2,\ 1\rangle$ *and* $\vec{r}\,(2) = \langle 0,\ 2,\ -1\rangle$.

Problem 3. *Find the domain of* $f(x,y) = \sqrt{4-x^2-y^2} + \ln(x^2y)$. *Then graph this domain.*

Problem 4. *Find the domain of* $f(x,y) = \sqrt{9-x^2-y^2} + \arcsin(x)$. *Then graph this domain.*

Problem 5. *Find the domain of* $f(x,y) = \sqrt{x^2+y^2-4} + \ln(4-y-x)$. *Then graph this domain.*

Problem 6. *Show that* $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^6+y^2} = 0$.

Problem 7. *Show that the following limit does not exist:* $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$.

Problem 8. *Show that the following limit does not exist:* $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x\sqrt{y}}{x^2+y}$.

Problem 9. *Find* $\dfrac{\partial^2 f}{\partial x \partial y}$, *and* $\dfrac{\partial^2 f}{\partial y \partial x}$ *when* $f(x,y) = x\sin y + \displaystyle\int_y^{2x} \sin(t^2)\,dt$.

Problem 10. *Find* $\dfrac{\partial^2 f}{\partial x \partial y}$, *and* $\dfrac{\partial^2 f}{\partial y \partial x}$ *when* $f(x,y) = x^2\cos y + \displaystyle\int_{2y}^{xy} \sqrt{1+t^4}\,dt$.

Problem 11. *Show that the function* $f(x,y) = x^3 - 3xy^2 + e^{3x}\cos 3y$ *is harmonic.*

Problem 12. *Find* $g'(t)$ *when* $g(t) = f(4\sin 2t,\ 5\cos 2t)$.

Problem 13. *Use differential (linear approximation) to estimate* $\sqrt[4]{16.03}\cdot\sin\left(\dfrac{7\pi}{20}\right)$.

Problem 14. *Use differential (linear approximation) to estimate* $\sqrt[4]{81.05}\cdot\sin\left(\dfrac{21\pi}{80}\right)$.

Problem 15. *Let* $f(x,y) = \sin x\,\cos y$. *Use the linear approximation to estimate* $f(\dfrac{9\pi}{20}, \dfrac{3\pi}{10})$.

Problem 16. *Find the direction in which* $f(x,y,z) = e^x + e^y - e^{2z}$ *increases most rapidly at* $(1,1,-1)$.

Problem 17. *Find the directional derivative of* $f(x,y,z) = x^2 + yz$ *at* $(1,-3,2)$ *in the direction of the path* $\mathbf{r}(t) = t^2\ \mathbf{i} + 3t\ \mathbf{j} + (1-t^3)\ \mathbf{k}$

Problem 18. *Use the chain rule to find* $\dfrac{du}{dt}$ *where* $u = x^2 - 3xy + 2y^2;\ x = \cos t,\ y = \sin t.$

Problem 19. *Use the chain rule to find* $\dfrac{\partial u}{\partial t},\ \dfrac{\partial u}{\partial s}$ *for* $u = x^2 - 3xy + 2y^2;\ x = s \cdot \cos t,\ y = t \cdot \sin s.$

Problem 20. *Write equation of tangent plane to the surface* $z = 3x^2y - xy^3$ *at the point* $P = (-1,\ 1,\ 4)$ *of this surface.*

Problem 21. *Find an equation for the tangent plane and symmetric equations for the normal line to the surface* $x^2z + xy^2 - z^3 = 5$ *at the point* $(2,-1,1)$.

Problem 22. *Find the critical points of* $f(x,y) = x^3 + y^3 - 3x^2 + 6y^2 + 3x + 12y + 7,$ *and test them for local maximum and minimum.*

Problem 23. *Find the absolute maximum and minimum of the function* $f(x,y) = x^2 + y^2 + x^2y + 4$ *on the set* $\{(x,y) :\ |x| \le 1,\ |y| \le 1\}$.

Problem 24. *Find the absolute maximum and minimum of the function* $f(x,y) = 2x - 2xy + y^2$ *over the region in* $xy-$*plane bounded by the graphs of* $y = x^2, y = 2$.

Problem 25. *Find the extreme values of* $f(x,y) = x^2 + 2y^2 - 2x + 3$ *subject to the constraint* $x^2 + y^2 \le 10$.

Problem 26. *Evaluate the integral* $\displaystyle\iint_R (4y + x)\, dA,$ *where* R *is the region between the parabola* $y = 3 - x^2$ *and* $y = 2x$.

Problem 27. *Evaluate* $\displaystyle\iint_R (4 + x^2)\, dA,$ *where* R *is the bounded region between the parabolas* $y = 1 + x^2,$ *and* $y = 3 - x^2$.

Problem 28. *Evaluate* $\displaystyle\iint_R x^2y\, dA,$ *where* R *is the region in the first quadrant of xy-plane bounded by* $y = x^2 - x$ *and* $y = 2$.

Problem 29. *Evaluate* $\displaystyle\int_0^2 \int_{x^2}^4 x\, e^{y^2}\, dy\, dx$

Problem 30. *Evaluate* $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} e^{(x^2+y^2)}\ dy\, dx.$

Problem 31. *Evaluate* $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2)\, dx\, dy.$