Review Problems For Test # 3

Problem 1. *Change the order of the following triple integral to $dx\,dz\,dy$:*

$$\int_0^4 \int_0^{2-x^2/8} \int_0^{1-x/4} f(x,y,z)\,dy\,dz\,dx$$

Problem 2. *Sketch the solid whose volume is given by the iterated integral*

$$\int_0^1 \int_0^{4(1-x)} \int_0^{2-y^2/8} dz\,dy\,dx$$

Problem 3. *Evaluate the integral* $\iiint_D xyz\,dV$, *where* D *is the solid region bounded below by xy-plane and above by the hemisphere* $z = \sqrt{9-x^2-y^2}$.

Problem 4. *Evaluate* $\iiint_D x\,dV$, *where* D *is the solid region in the first octant bounded by the coordinate planes with* $x^2+y^2+z^2 \le 9$.

Problem 5. *Evaluate* $\iiint_D (1+z^2)\,dV$, *where* D *is the solid region bounded below by the upper portion of the cone* $z^2 = 3x^2+3y^2$ *and above by the sphere* $x^2+y^2+z^2 = 4$.

Problem 6. *Find the volume of the solid region in the first octant bounded by the paraboloid* $z = x^2+y^2$ *and the upper portion of the cone* $z^2 = x^2+y^2$.

Problem 7. *Find the volume of the solid region in the first octant bounded by the cylinder* $x^2+y^2 = 9$, *the plane* $z = 3y$, *the* $xy-$*plane, and the* $yz-$*plane.*

Problem 8. *Evaluate the following integral, by using a suitable change of variables* $\iint_R (2x-y^2)dx\,dy$, *where* R *is the region bounded by the lines* $x-y=1$, $x-y=3$, $x+y=2$, *and* $x+y=4$.

Problem 9. *Evaluate the following integral, by using a suitable change of variables* $\iint_R \cos\left(\frac{y-x}{y+x}\right)dA$, *where* R *is the region bounded by the lines* $x+y=2$, $x+y=4$, $x=0$, *and* $y=0$.

Problem 10. *Evaluate the following* $\iint_R \frac{16xy(x^2-y^2)}{x^2+y^2}dA$, *where* R *is the region in the first quadrant bounded by the curves* $x^2+y^2=4$, $x^2+y^2=9$, $x^2-y^2=1$, *and* $x^2-y^2=4$.

Problem 11. *Evaluate the integral* $\iint_R (x^2+y^2)(x^2-y^2)\,dA$, *where* R *is the region in the first quadrant bounded by the lines* $y=x$, $y=x-1$, *and circles* $x^2+y^2=1$, *and* $x^2+y^2=9$.

Problem 12. *Find the area of the region in the first quadrant bounded by* $x^2-y^2=1$, $x^2-y^2=4$, $x+y=2$, *and* $x+y=3$.

Problem 13. *Evaluate the integral* $\iint_R (x+y)^2(x-y)\,dA$, *where* R *is the region in the first quadrant bounded by* $x^2-y^2=1$, $x^2-y^2=4$, $x+y=2$, *and* $x+y=3$.

Problem 14. *Is the vector field* $\overrightarrow{\mathbf{F}}(x,y)=\langle 3x^2y^2+10x,\ 2x^3y+4\rangle$ *conservative? If it is, find a potential function for* $\overrightarrow{\mathbf{F}}$.

Problem 15. *Is the vector field* $\overrightarrow{\mathbf{F}}(x,y)=(\ y^2\,e^{xy}\ )\ \vec{\imath}+(1+xy)e^{xy}\ \vec{\jmath}$ *conservative? If it is, find a potential function for* $\overrightarrow{\mathbf{F}}$.

Problem 16. *Is the vector field* $\overrightarrow{\mathbf{F}}(x,y,z)=\left\langle y^2e^z+10x,\, 2xye^z-6z^2,\, xy^2e^z-12yz\right\rangle$ *conservative? If it is, find a potential function for* $\overrightarrow{\mathbf{F}}$.

Problem 17. *Evaluate the line integral* $\int_C (2x+3y)\,ds$, *where* C *is the portion of the circles* $(x-1)^2+y^2=1$ *from* $(0,0)$ *to* $(2,0)$.

Problem 18. *Evaluate the line integral* $\int_C (2x+3y)\,dx$, *where* C *is the portion of the circles* $(x-1)^2+y^2=4$ *in the first quadrant from* $(3,0)$ *to* $(0,\sqrt{3}\,)$.

Problem 19. *Evaluate the line integral* $\int_C \overrightarrow{\mathbf{F}}\cdot d\vec{r}$, *where* C *is the portion of the circles* $(x-1)^2+y^2=1$ *from* $(1,1)$ *to* $(2,0)$ *clockwise and* $\overrightarrow{\mathbf{F}}(x,y)=\langle y,\ x\rangle$.

Problem 20. *Evaluate the line integral* $\int_C \overrightarrow{\mathbf{F}}\cdot d\vec{r}$, *where* C *is the portion of the circles* $(x-1)^2+y^2=1$ *from* $(1,-1)$ *to* $(1,1)$ *counter-clockwise and* $\overrightarrow{\mathbf{F}}(x,y)=\langle 3x^2y+2x,\ x^3\rangle$.

Problem 21. *Evaluate the line integral* $\int_C \overrightarrow{\mathbf{F}}\cdot d\vec{r}$, *where* C *is the circles* $x^2+y^2=1$ *and* $\overrightarrow{\mathbf{F}}(x,y)=\langle 4x^3y^2-3y,\ 2x^4y\rangle$.

Problem 22. *Evaluate the line integral* $\int_C \overrightarrow{\mathbf{F}}\cdot d\vec{r}$, *where* C *is the triangle formed by the lines* $3x+y=3$, $x=0$, $y=0$, *and* $\overrightarrow{\mathbf{F}}(x,y)=\langle 4x^3y^2+3y,\ 2x^4y+5x\rangle$.

Problem 23. *Evaluate the line integral* $\int_C \overrightarrow{\mathbf{F}}\cdot d\vec{r}$, *where* C *is the closed curve consists of semi-circle* $x^2+y^2=1$ $(y\geq 0)$ *and the portion of* x*-axis joining* $(-1,0)$ *to* $(1,0)$ *traversed counterclockwise, and* $\overrightarrow{\mathbf{F}}(x,y)=\langle 2xe^{y^2}-3y^4,\ 2x^2ye^{y^2}+4x^3y\rangle$.